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THE CENTROID OF AREAS AND VOLUMES.

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It is the object of this paper to put on record, once for all, general values for the centroid of areas, represented by the curve $\left(\frac{x}{a}\right)^{\frac{2}{2m+1}} + \left(\frac{y}{b}\right)^{\frac{2}{2n+1}} = 1$, and the centroid of volumes represented by the surface

$$\left(\frac{x}{a}\right)^{\frac{2}{2m+1}} + \left(\frac{y}{b}\right)^{\frac{2}{2n+1}} + \left(\frac{z}{c}\right)^{\frac{2}{2p+1}} = 1.$$

I. *Areas.* Let the density vary as $x^{k-1}y^{l-1}$, the thickness being constant.

$$\text{Then } \bar{x} = \frac{\iint x^k y^{l-1} dx dy}{\iint x^{k-1} y^{l-1} dx dy}, \quad \bar{y} = \frac{\iint x^{k-1} y^l dx dy}{\iint x^{k-1} y^{l-1} dx dy}.$$

$$\begin{aligned} \bar{x} &= \frac{a^{k+1}b^l}{\frac{4}{(2m+1)(2n+1)} I' \left\{ \frac{k+1}{2}(2m+1) \right\} I' \left\{ \frac{l}{2}(2n+1) \right\}} \\ &= \frac{a^k b^l}{\frac{4}{(2m+1)(2n+1)} I' \left\{ \frac{k}{2}(2m+1) \right\} I' \left\{ \frac{l}{2}(2n+1) \right\}} \\ &= \frac{a^k b^l}{\frac{4}{(2m+1)(2n+1)} I' \left\{ \frac{k}{2}(2m+1) + \frac{l}{2}(2n+1) + 1 \right\}}. \end{aligned}$$

$$\begin{aligned} \bar{x} &= \frac{I'(km+m+\frac{k+1}{2}) I'(kn+ln+\frac{k+l}{2}+1)}{I'(km+\frac{k}{2}) I'(kn+ln+m+\frac{k+l+1}{2}+1)} a \dots \dots \dots (A). \end{aligned}$$

$$\begin{aligned} \text{Similarly, } \bar{y} &= \frac{I'(ln+n+\frac{l+1}{2}) I'(km+ln+\frac{k+l}{2}+1)}{I'(ln+\frac{l}{2}) I'(km+ln+n+\frac{k+l+1}{2}+1)} b \dots \dots \dots (B). \end{aligned}$$

This gives the centroid of a quadrant of the area whatever be the values of k, l, m, n . Let $k=l=1$, so that the density is the same throughout the whole area.

$$\therefore \bar{x} = \frac{\Gamma(2m+1)\Gamma(m+n+2)}{\Gamma(m+\frac{1}{2})\Gamma(2m+n+\frac{5}{2})}a, \quad \bar{y} = \frac{\Gamma(2n+1)\Gamma(m+n+2)}{\Gamma(n+\frac{1}{2})\Gamma(m+2n+\frac{5}{2})}b.$$

Let $m=n=0$, then $\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$.

$$\therefore \bar{x} = \frac{\Gamma(1)\Gamma(2)}{\Gamma(\frac{1}{2})\Gamma(\frac{5}{2})}a = \frac{4a}{3\pi}, \quad \bar{y} = \frac{\Gamma(1)\Gamma(2)}{\Gamma(\frac{1}{2})\Gamma(\frac{5}{2})}b = \frac{4b}{3\pi}.$$

Let $m=n=1$, then $\left(\frac{x}{a}\right)^{\frac{3}{2}} + \left(\frac{y}{b}\right)^{\frac{3}{2}} = 1$.

$$\therefore \bar{x} = \frac{\Gamma(3)\Gamma(4)}{\Gamma(\frac{3}{2})\Gamma(\frac{11}{2})}a = \frac{256a}{315\pi}, \quad \bar{y} = \frac{\Gamma(3)\Gamma(4)}{\Gamma(\frac{3}{2})\Gamma(\frac{11}{2})}b = \frac{256b}{315\pi}.$$

Let $m=n=2$, then $\left(\frac{x}{a}\right)^{\frac{5}{2}} + \left(\frac{y}{b}\right)^{\frac{5}{2}} = 1$.

$$\therefore \bar{x} = \frac{\Gamma(5)\Gamma(6)}{\Gamma(\frac{5}{2})\Gamma(\frac{17}{2})}a = \frac{2.4.8.12.16.20}{3.5.7.9.11.13.15} \cdot \frac{4a}{\pi},$$

$$\bar{y} = \frac{\Gamma(5)\Gamma(6)}{\Gamma(\frac{5}{2})\Gamma(\frac{17}{2})}b = \frac{2.4.8.12.16.20}{3.5.7.9.11.13.15} \cdot \frac{4b}{\pi}.$$

Let $m=0, n=1$, then $\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^{\frac{3}{2}} = 1$.

$$\therefore \bar{x} = \frac{\Gamma(1)\Gamma(3)}{\Gamma(\frac{1}{2})\Gamma(\frac{7}{2})}a = \frac{16a}{15\pi}, \quad \bar{y} = \frac{\Gamma(3)\Gamma(3)}{\Gamma(\frac{3}{2})\Gamma(\frac{7}{2})}b = \frac{128b}{105\pi}.$$

Let $m=n=\frac{3}{2}$, then $\left(\frac{x}{a}\right)^{\frac{1}{2}} + \left(\frac{y}{b}\right)^{\frac{1}{2}} = 1$.

$$\therefore \bar{x} = \frac{\Gamma(4)\Gamma(5)}{\Gamma(2)\Gamma(7)}a = \frac{a}{5}, \quad \bar{y} = \frac{\Gamma(4)\Gamma(5)}{\Gamma(2)\Gamma(7)}b = \frac{b}{5}, \text{ the centroid of the area be-}$$

tween the parabola and its tangents as axes.

Let the density vary as xy , so that $k=l=2$.

$$\therefore \bar{x} = \frac{\Gamma(3m+\frac{3}{2})\Gamma(2m+2n+3)}{\Gamma(2m+1)\Gamma(3m+2n+\frac{7}{2})}a, \quad \bar{y} = \frac{\Gamma(3n+\frac{3}{2})\Gamma(2m+2n+3)}{\Gamma(2n+1)\Gamma(2m+3n+\frac{7}{2})}b.$$

Let $m=n=0$, then $\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$.

$$\therefore \bar{x} = \frac{\Gamma(\frac{3}{2})\Gamma(3)}{\Gamma(1)\Gamma(\frac{7}{2})}a = \frac{8a}{15}, \bar{y} = \frac{\Gamma(\frac{3}{2})\Gamma(3)}{\Gamma(1)\Gamma(\frac{7}{2})}b = \frac{8b}{15}.$$

$$\text{Let } m=n=1, \text{ then } \left(\frac{x}{a}\right)^{\frac{8}{3}} + \left(\frac{y}{b}\right)^{\frac{8}{3}} = 1.$$

$$\therefore \bar{x} = \frac{\Gamma(\frac{9}{2})\Gamma(7)}{\Gamma(3)\Gamma(\frac{17}{2})}a = \frac{128a}{429}, \bar{y} = \frac{\Gamma(\frac{9}{2})\Gamma(7)}{\Gamma(3)\Gamma(\frac{17}{2})}b = \frac{128b}{429}.$$

$$\text{Let } m=n=\frac{3}{2}, \text{ then } \left(\frac{x}{a}\right)^{\frac{1}{2}} + \left(\frac{y}{b}\right)^{\frac{1}{2}} = 1.$$

$$\therefore \bar{x} = \frac{\Gamma(6)\Gamma(9)}{\Gamma(4)\Gamma(11)}a = \frac{2a}{9}, \bar{y} = \frac{\Gamma(6)\Gamma(9)}{\Gamma(4)\Gamma(11)}b = \frac{2b}{9}.$$

Let the density vary as x the distance from the axis of ordinates so that $k=2, l=1$.

$$\therefore \bar{x} = \frac{\Gamma(3m+\frac{3}{2})\Gamma(2m+n+\frac{5}{2})}{\Gamma(2m+1)\Gamma(3m+n+3)}a, \bar{y} = \frac{\Gamma(2n+1)\Gamma(2m+n+\frac{5}{2})}{\Gamma(n+\frac{1}{2})\Gamma(2m+2n+3)}b.$$

$$\text{Let } m=n=0, \text{ then } \left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1.$$

$$\therefore \bar{x} = \frac{\Gamma(\frac{3}{2})\Gamma(\frac{5}{2})}{\Gamma(1)\Gamma(3)}a = \frac{3\pi a}{16}, \bar{y} = \frac{\Gamma(1)\Gamma(\frac{5}{2})}{\Gamma(\frac{1}{2})\Gamma(3)}b = \frac{3b}{8}.$$

$$\text{Let } m=n=1, \text{ then } \left(\frac{x}{a}\right)^{\frac{8}{3}} + \left(\frac{y}{b}\right)^{\frac{8}{3}} = 1.$$

$$\therefore \bar{x} = \frac{\Gamma(\frac{9}{2})\Gamma(\frac{11}{2})}{\Gamma(3)\Gamma(7)}a = \frac{49.45\pi a}{2^{14}}, \bar{y} = \frac{\Gamma(3)\Gamma(\frac{11}{2})}{\Gamma(\frac{3}{2})\Gamma(7)}b = \frac{63b}{384}.$$

$$\text{Let } m=n=\frac{3}{2}, \text{ then } \left(\frac{x}{a}\right)^{\frac{1}{2}} + \left(\frac{y}{b}\right)^{\frac{1}{2}} = 1.$$

$$\therefore \bar{x} = \frac{\Gamma(6)\Gamma(7)}{\Gamma(4)\Gamma(9)}a = \frac{5a}{14}, \bar{y} = \frac{\Gamma(4)\Gamma(7)}{\Gamma(2)\Gamma(9)}b = \frac{3b}{28}.$$

Let the density vary as y the distance from the axis of abscissas so that $k=1$, $l=2$.

$$\therefore \bar{x} = \frac{\Gamma(2m+1)\Gamma(m+2n+\frac{1}{2})}{\Gamma(m+\frac{1}{2})\Gamma(2m+2n+3)}a, \quad \bar{y} = \frac{\Gamma(3n+\frac{3}{2})\Gamma(m+2n+\frac{5}{2})}{\Gamma(2n+1)\Gamma(m+3n+3)}b.$$

Let $m=n=0$, then $\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$.

$$\therefore \bar{x} = \frac{\Gamma(1)\Gamma(\frac{1}{2})}{\Gamma(\frac{1}{2})\Gamma(3)}a = \frac{3a}{8}, \quad \bar{y} = \frac{\Gamma(\frac{3}{2})\Gamma(\frac{5}{2})}{\Gamma(1)\Gamma(3)}b = \frac{3\pi b}{16}.$$

Let $m=n=1$, then $\left(\frac{x}{a}\right)^{\frac{2}{3}} + \left(\frac{y}{b}\right)^{\frac{2}{3}} = 1$.

$$\therefore \bar{x} = \frac{\Gamma(3)\Gamma(\frac{11}{2})}{\Gamma(\frac{3}{2})\Gamma(7)}a = \frac{63a}{384}, \quad \bar{y} = \frac{\Gamma(\frac{5}{2})\Gamma(\frac{11}{2})}{\Gamma(3)\Gamma(7)}b = \frac{49.45\pi b}{2^{14}}.$$

Let $m=n=\frac{3}{2}$, then $\left(\frac{x}{a}\right)^{\frac{1}{2}} + \left(\frac{y}{b}\right)^{\frac{1}{2}} = 1$.

$$\therefore \bar{x} = \frac{\Gamma(4)\Gamma(7)}{\Gamma(2)\Gamma(9)}a = \frac{3a}{28}, \quad \bar{y} = \frac{\Gamma(6)\Gamma(7)}{\Gamma(4)\Gamma(9)}b = \frac{5b}{14}.$$

[To be Continued.]